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This has been an especially fruitful year for several long-range projects. All of them of central importance in fractal geometry, and this grant from ONR has supported them for several years. My work has been rewarded by the 1993 *Wolf Prize for Physics* and the 1994 *Honda Foundation Prize*.

STUDY OF DLA: Additional tests have confirmed that in two dimensions the clusters of DLA (the diffusion limited aggregates) are *not* self similar. A substitute improved description of DLA was provided.

• If DLA were self-similar, the properly renormalized moments would be independent of the cluster size N. Therefore, the observed values have been expected to convey to a limit. To the contrary, as we reported at a meeting held in Budapest in 1993, these relative moments have *not* stabilized by $N \sim 1$ to 10 million atoms. Given the controversial nature of this finding, we tested it again and again, then replotted in ways meant to find flaws in the conclusion. Not only does the conclusion hold, but it is reinforced.

• If DLA were self-similar, the fjords would widen proportionately to the cluster radius N, and the largest fjords' widths (as seen from the origin and measured of degrees) would be independent of N. To the contrary, we have found that those widths in degrees clearly continue to decrease as N grows and are far from having stabilized by $N \sim 1$ to 10 million atoms.

• I had argued at a meeting held in Hamburg in 1992 that the fractal dimension near a point P of the cluster should be smaller if measured in a direction perpendicular to the radius of OP. This prediction has been strongly verified. We show that it is not a symptom of self-affinity, but of varying lacunarity.—

• We defined "Parallel DLA" as the cluster obtained when an existing cluster is hit simultaneously by $M = 32$ atoms instead of $M = 1$. The result is not DLA but the kind of cluster expected in an environment of positive density. This process was investigated in detail.

• All earlier computer studies of the harmonic measure, especially around DLA, were massed by large numerical biases. A new method

for very accurate evaluation of the harmonic measure was developed and is being used to either confirm or invalidate the story anomalies reported by Mandelbrot and Evertsz in 1991.

- A demanding and careful comparison of the cylindrical and circular versions of DLA is nearly completed

- It was shown that notwithstanding all the observed departures from self-similarity, DLA clusters are locally isotropic, i.e., show no trace of a direction of growth.

STUDY OF LACUNARITY: Lacunarity is a long-held interest of the P. I. He has shown that like "bigness," it is not a property quantifiable by one number but a group of related but distinct properties that require several numbers

The P. I. developed a new concept of lacunarity as measured by the "antipodal correlation" between the masses located in opposite spatial cones with a point in a fractal as their common apex. When those "antipodal cones" are statistically independent, a point shields two opposite directions from each other and lacunarity is said to be neutral. Positive correlation measures below-neutral lacunarity and negative correlation measures above neutral lacunarity

Working with D. Stauffer, the P. I. has shown that critical percolation clusters in the plane are of neutral lacunarity

An older notion of lacunarity is one based on the ϵ neighborhoods. It was evaluated for a new large and the important class of fractals.

FRACTAL SUMS OF PULSES (FSP): The FSP form a new class of random functions of surprising by varied form and great promise in modeling. They are defined as generalizations of Lévy flights

A Lévy flight is the sum of an infinity of step-functions of widely varying sizes. We have generalized the Lévy construction by replacing the jumps by suitable affine reductions or dilations of kernel functions more general than a step. The result can be described as being an *affine convolution*. A kernel that is constant except on a (bounded) interval, will be called *pulse*, and resulting sums will be called *fractal sums of pulses* (FSP). This paper describes the theory of some self-affine FSP and some concrete applications

A pulse's height and location still follow the same distribution as in a Lévy flight: the probability of the point $[\lambda, t]$ being found in an elementary rectangle of the $[\lambda, t]$ address plane is $\propto \lambda^{-\delta-1} d\lambda dt$. A major immediate difference is that in a FSP δ is not always constrained to satisfy $0 < \delta < 2$. In a semi-random self-affine FSP, the pulse's width w (the length of the smallest interval in which the pulse varies) is $w = a\lambda^{t}$ where $a > 0$ is a scale constant, implying that $Pr[W > w] \propto w^{-\theta}$ is statistically independent of λ with the measure $\propto w^{-\theta-1} dw$ where $0 < \theta < 1$. The pulse forms we have studied so far exemplify a variety of distinct behaviors: discontinuous versus continuous, canceling (vanishing outside of the interval in which they vary), versus non-canceling. Examples include cylinders (one

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discontinuous *rise* followed after the time w by one discontinuous *fall*), cones (uniform rate rise followed by uniform rate fall), and multiple steps.

Being self-affine, all FSP are not changed by the rescaling that is ordinarily used to describe attraction of a process to a limit such as Brownian motion. (In physicists' language, each FSP defines its own "class of universality" with respect to a suitable affinity.) But under an alternative form of rescaling, each semi-random FSP falls in a standard domain of attraction. The attractor may be a Lévy flight, which is not a great surprise, because one starts with the Lévy measure. But it may be a fractional Brownian motion, which is a surprise and establishes deep new links between two independently arisen theories that are known to have striking formal parallelisms.

We have also investigated diverse classes of processes related to the FSP.

A) Semi random FSP with $w = a\lambda^{\theta}$ where $\theta \neq \delta$; they are not self-affine.

B) Fully random FSP in which λ and W are naturally independent; they are self-affine with $H = (1 - \theta)/\delta$.

C) Fractal sums of *micropulses* (FSM); they generate fractional Brownian motions.